AIR DENSITY AND ITS UNCERTAINTY

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Air and its composition

The air is a mixture of several gases → dry air, and water in steam form. Troposphere is the inferior layer of the terrestrial atmosphere, terrestrial surface altitude of 6 to 18 kilometers, the air we breathed is concentrated there.

The dry air as the water steam behaves like ideal gases. They have been developed to empirical laws that relate the macroscopic values, in ideal gases, these values include pressure (p), volume (V) and temperature (T)

Ley de Charles → IDEAL GAS LAW

Ley de Gay-Lussac → IDEAL GAS LAW

Ley de Boyle → IDEAL GAS LAW

IDEAL GAS LAW

$$\frac{P_1}{T_1} = \frac{P_2}{T_2} \quad \wedge \quad \frac{V_1}{T_1} = \frac{V_2}{T_2} \quad \wedge \quad P_1.V_1 = P_2V_2 \quad \Rightarrow \quad \frac{P_1.V_1}{T_1} = \frac{P_2V_2}{T_2}$$

Air and its composition

It is constituted by a nitrogen mixture and of oxygen like basic element (99%) and the rest like noble gases. The composition is similar around the world.

- Water Steam (0-5%), Carbon dioxide, hydrocarbons, tars, ashes, dust and SO2.
- Electrical delivery form C2H2, H202, 03, NO3H, NH3, NO3NH4.

	COMPOSICION DEL AIRE PURO	
	Proporción	Proporción
Elemento	en volumen	en peso
Nitrógeno	78,14	75,6
Oxigeno	20,92	23,1
Argón	0,94	0,3
Neón	$1{,}5\ 10^{-3}$	$1\ 10^{-3}$
Helio	$5\ 10^{-4}$	$0{,}7\ 10^{-4}$
Criptón	$1\ 10^{-4}$	$3\ 10^{-4}$
Hidrogeno	$5\ 10^{-5}$	$0{,}35\ 10^{-5}$
Xenón	$1\ 10^{-5}$	$4\ 10^{-5}$

Ways to calculate the air density

Density defined in a qualitative manner as the measure of the relative mass of objects with a constant volume

$$\delta = \frac{m}{V}$$

Hypothesis de Avogadro Two gases same volume (same pressure and temperature) contain the same number of particles, or molecules

Standard Law gases

P.V = n . R . T   n = m/Mr

$$P.V = \frac{m}{Mr} R.T \Rightarrow P.Mr = \underbrace{\frac{m}{V}}_{\delta}.R.T \Rightarrow \boxed{P.Mr = \delta.R.T}$$

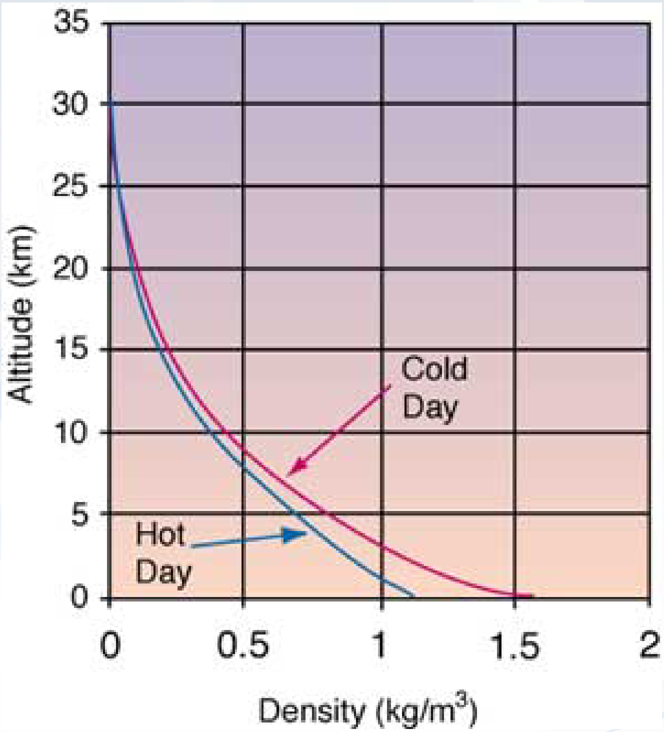
Ways to calculate the air density

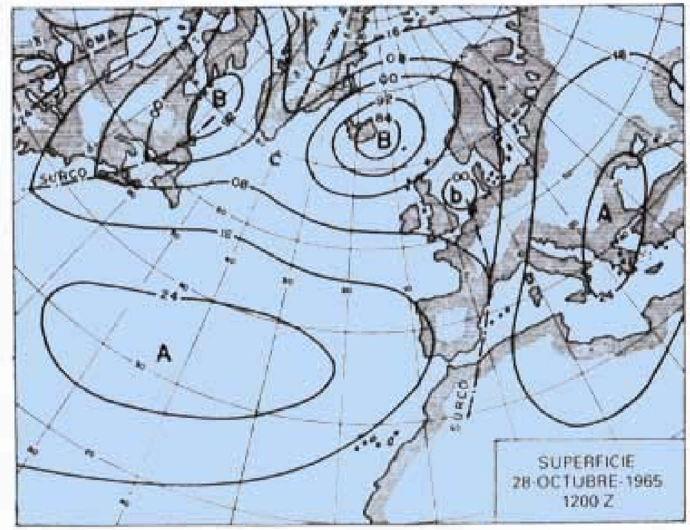
- As function of altitude
- Using a refractometer
- Air buoyancy artefacts methods
- Equation CIPM /81
- Approximate equation

Ways to calculate the air density

AS FUNCTION OF ALTITUDE

Atmospheric pressure drops about or about 1.1 mbar (kPa) for each 100 meters. **Density decreases**





Representación del campo de presión en superficie.

Ways to calculate the air density AS FUNCTION OF ALTITUDE

L = 6,5 temperature lapse rate, deg K/km

H = geopotencial altitude

Z = geometrical altitude

To = Temperature °K

Po = Atmospheric Pressure

$$T = T_0 - L \cdot H$$

$$P = P_0 * \left(1 - \frac{L * H}{T_0}\right)^{\frac{g*M}{R*L}}$$

$$Z = \frac{E * H}{E - H}$$

$$D = \frac{P * M}{R * T * 1000}$$

Ways to calculate the air density REFRACTROMETRY

Changes in air density can be determined with good precision using an optical method based on the high correlation between air density and air index of refraction.

$$\rho = \frac{2}{3R'}(n - 1),$$

R specific refraction or the refractional invariant in unction composition of air and the local atmospheric conditions

$$n = \frac{\nu_{vacio}}{\nu_{aire}}$$

n is determined by a simple ratio of laser frequencies:

ν_{vacio} laser frequency locked to one transmission peak of the interferometer under vacuum

ν_{aire} the frequency locked to the same peak of the interferometer placed in air.

Ways to calculate the air density

AIR BUOYANCY ARTEFACTS METHODS

The method is based on the weighing of two artefacts having the same nominal mass and the same surface area but with very different volumes. Two weightings are necessary to determine the air density ρ , one in air and one in vacuum

$$\Delta m_{aire} = I_1 - I_2 + \rho(V_{m1} - V_{m2})$$

I_1 e I_2 balance readings in air mass $_1$ and mass$_2$

V_{m1} e V_{m2} volume of m $_1$ y m $_2$

- ρ air density

$$\Delta m_{vacío} = I_3 - I_4$$

I_3 e I_4 the balance readings in vacuum mass $_1$ y mass $_2$

$$\Delta m_{aire} = \Delta m_{vacio} + \sigma \Delta S$$

- ΔS the difference in surface area between the two artefacts and σ mass of adsorption per unit area.

$$\rho = \frac{I_3 - I_4 - (I_1 - I_2 - \sigma \Delta S)}{V_{m1} - V_{m2}}$$

Ways to calculate the air density
FORMULA CIPM

From the equation of state of a non-ideal gas and the experimental conditions the density of moist air

$$\rho_a = \frac{pM_a}{ZRT}\left[1 - x_v\left(1 - \frac{M_v}{M_a}\right)\right]$$

$$M_a = [28,9635 + 12,011(x_{co_2} - 0,0004)] * 10^{-3}\, kgmol^{-1}$$

Where $$x_v = hf(p,t)\frac{p_{sv}(t)}{p}$$

P pressure

T thermodynamic temperature 273,15 + t

Mv molar mass of the water

Z compressibility factor

R molar gases constant

Ways to calculate the air density
APPROXIMATE EQUATION

From BIPM formula we obtain one numerical approximate equation :

$$\rho_a = \frac{0{,}34848 * p - 0{,}009024 * h_r * e^{0{,}0612 * t}}{273{,}15 + t}$$

Ways to calculate the air density
PSYCHROMETRY

Thermodynamic properties of mixtures of gas with vapor. saturation pressure and temperature of dew, Indexes of humidity, Volume, heat and humid enthalpy, temperature of saturation adiabatic and wet thermometer.

Some definitions:

Relative Humidity. The relative humidity is the percent of saturation humidity, generally calculated in relation to saturated vapor density, in (%):

$$HR = 100\ Pv/Ps\ (\%)$$

Temperature of adiabatic saturation, Th , is the ideal temperature of equilibrium will have the air non saturated after undergoing an adiabatic and isobaric process (iso enthalpic), that it takes it temperature to the saturation by means of liquid evaporation of water to this.

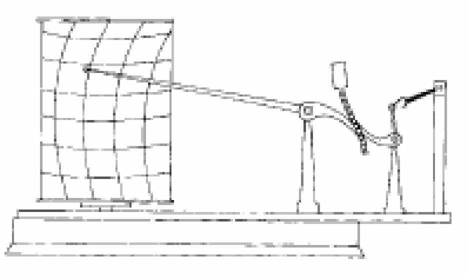
Temperature of wet thermometer is the temperature **that it reaches a thermometer covered with a wet cloth that is exposed to an airflow without saturating that it flows at speeds near 5 m/s**

Dew point is the temperature, at which the moisture content in the air will saturate the air , If the air is cooled further, some of the moisture will condense .

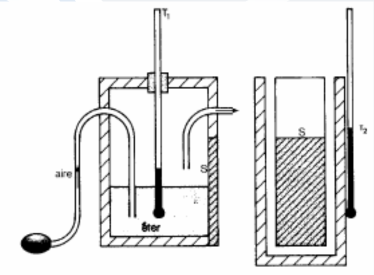
Ways to calculate the air density
PSYCHROMETRY

To mesure the humidity :

ARTEFACT	METHOD
psychrometer	Thermodynamic
hygrometer of hair or others materials	Hygroscopic
Hygrometer of dew point	Condensation
Hygrometer of Chemist absorption	Gravimetric
Hygrometer digital	Variation of electrical properties

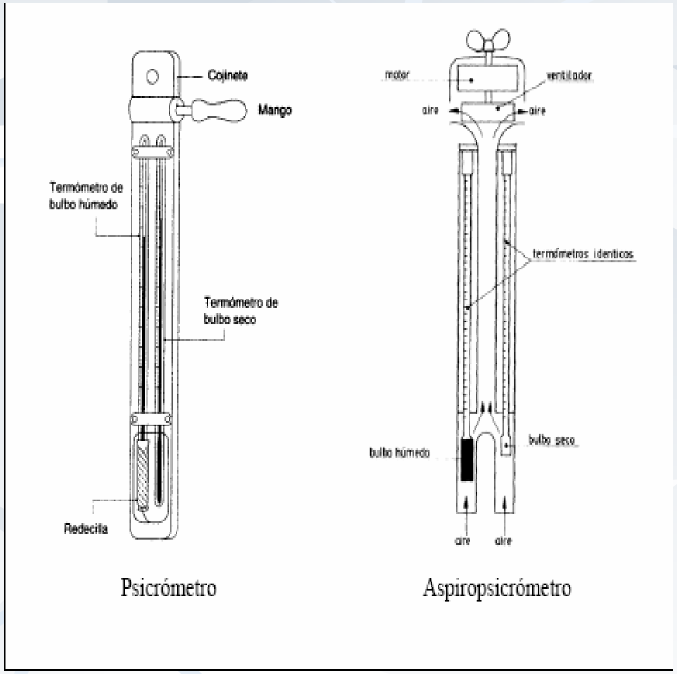


Higrógrafo de cabello



Esquema del higrómetro de punto de rocío.

Ways to calculate the air density PSYCHROMETRY

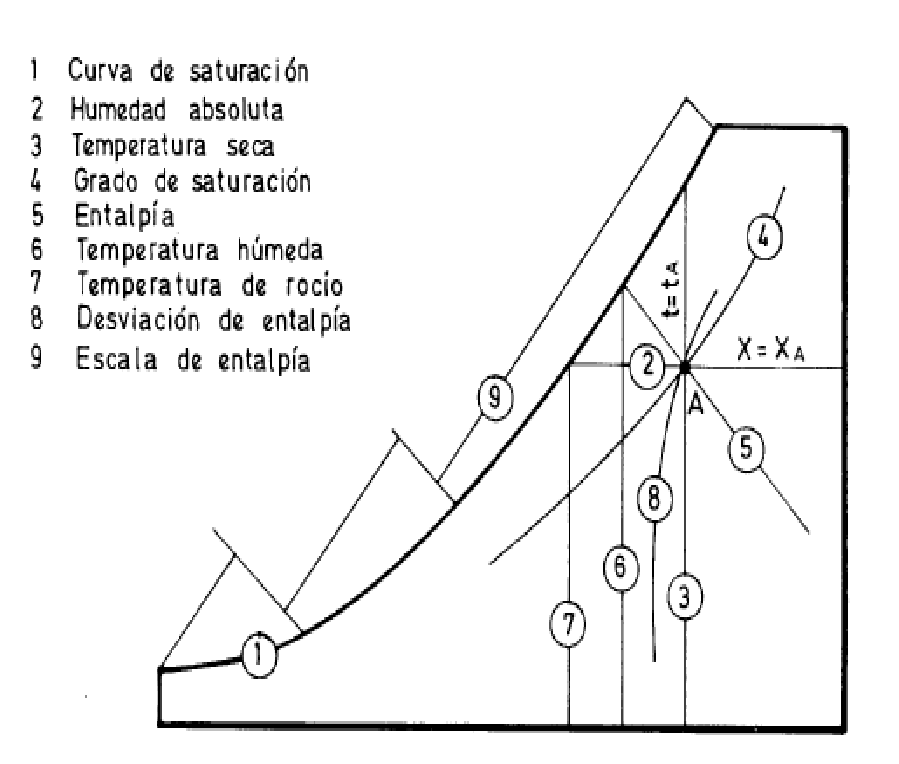


Psychrometer and aspiro psychrometer Consist two thermometers, one normal (dry) and another with their bulb permanently humidified thanks to a cloth or wet gauze that it recovers it.
The humidity can be measured between both starting from the difference of temperature apparatuses

Ways to calculate the air density PSYCHROMETRY

Diagram Carrier.

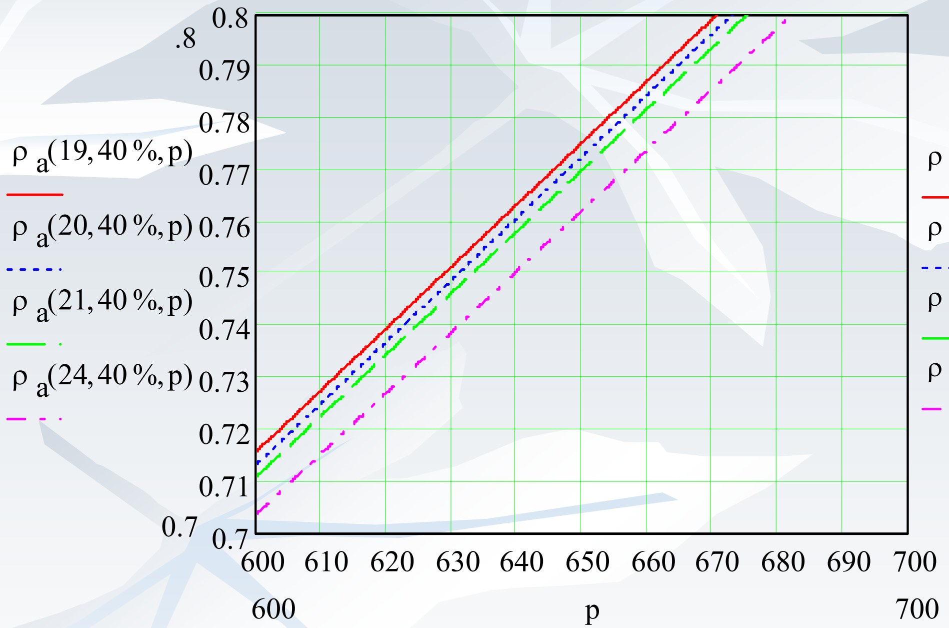
- The T represents (°C) in the abscissa axis (axis x) and the mixture reason or humidity (X, in kg of water/kg of dry air) in the axis of orderly (axis and, to the right).
- The saturation curve (HR = 100%) it ascends toward the right and it represents the end of the diagram. In this curve the temperatures of humid thermometer and the temperatures of dew are located.
- - The curves of humidity relative constant are similar to that of saturation, advancing down (lying down more) as it diminishes the humidity of the air.

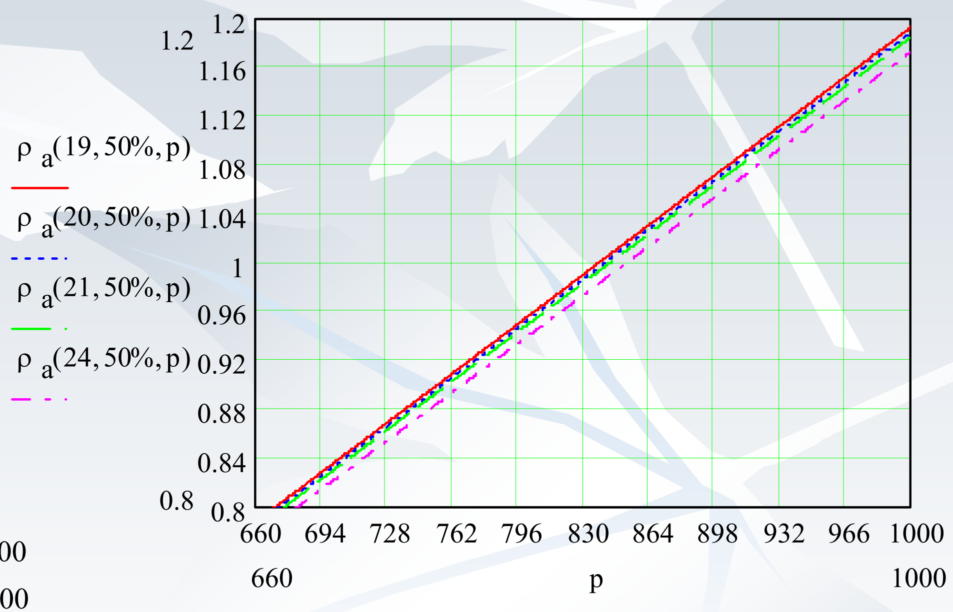


Esquema del diagrama Carrier

Ways to calculate the air density

CHART





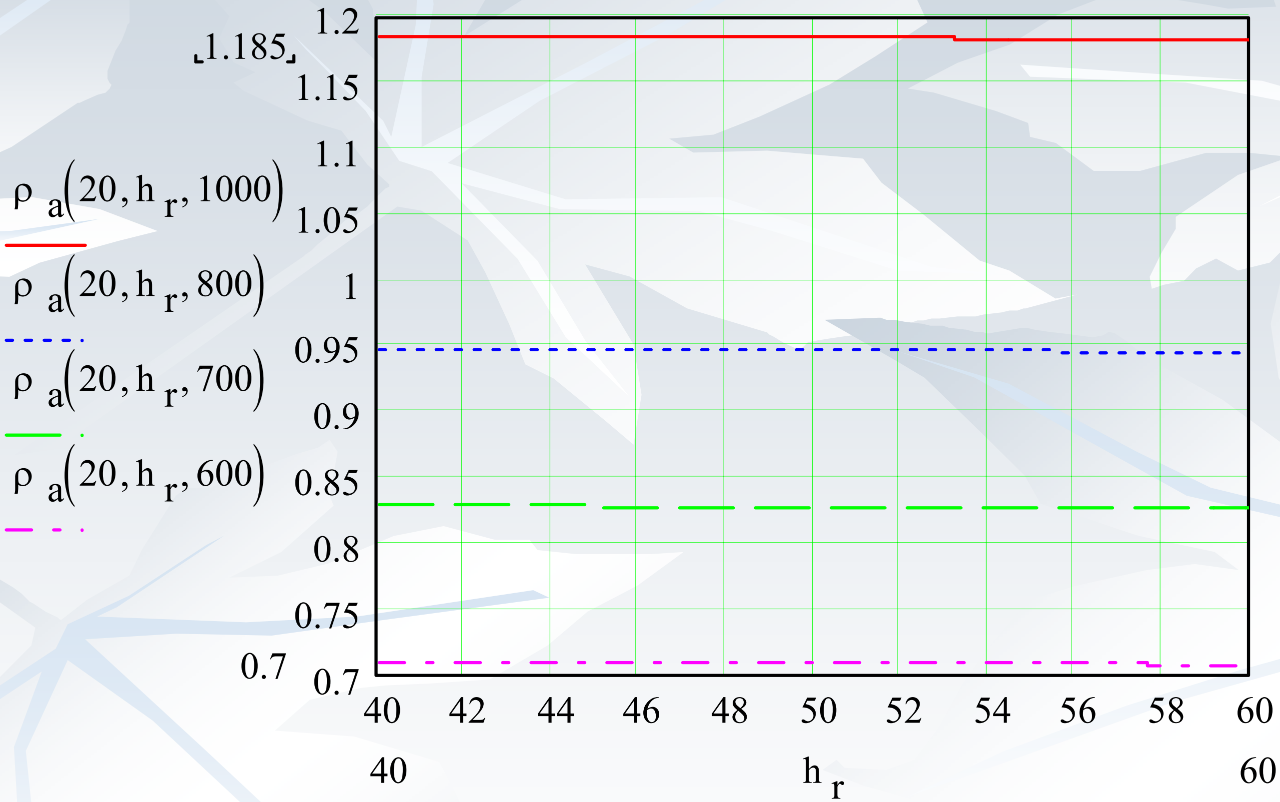
Air density evaluated with

Relative humidity 40 %,
temperature 19 °C – 24 °C,
pressure 600 mbar – 700 mbar

Air density with evaluated with
Relative humidity 50 %,
temperature 19 °C – 24 °C,
pressure 660 mbar – 1000 mbar

Ways to calculate the air density

CHART



Air density evaluated with

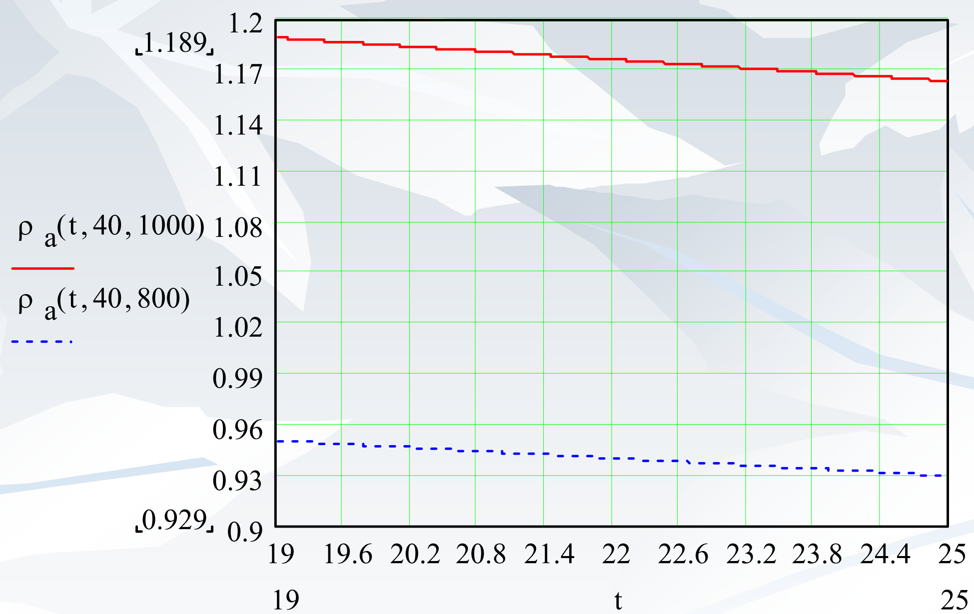
Relative humidity 40 % - 60 %,

temperature 20 °C ,

pressure 600 mbar – 1000 mbar

Ways to calculate the air density

CHART



Air density evaluated with

Relative humidity 40 %,

temperature 19 °C – 25 °C,

pressure 800 mbar – 1000 mbar

CALCULATION OF THE AIR DENSITY CIPM

$$\frac{pM_a}{ZRT}\left[1-x_v\left(1-\frac{M_v}{M_a}\right)\right]$$

Where:

p	**Pressure of air in Pa.**
M_a	**molar mass of dry air.**
Z	**Compressibility factor**
R	**Universal constant of ideal gases**
T	**Temperature of air in K**
X_v	**molar fraction of water steam**
M_v	**molar mass of water**

- **Molar mass of dry air, Ma**

If it considers constant of air component

$M_a = 0{,}028963\ 512440\ kg \cdot mol^{-1}$

If it can measure the concentration of CO_2

$M_a = [28{,}9635 + 12{,}011\ (X_{CO2} \cdot - 0{,}0004)] * 10^{-3}\ kg \cdot mol^{-1}$

- **Compressibility factor, Z**

$$Z = 1 - \frac{p}{T}\left[a_0 + a_1 t + a_2 t^2 + (b_0 + b_1 t)x_v + (c_0 + c_1 t)x_v^2\right] + \frac{p^2}{T^2}\left(d + e x_v^2\right)$$

Where:

p	Air pressure in Pa
T	Air temperature in K
t	Environmental temperature in ^oC
a_0	1, 581 23 *X* 10^{-6} K Pa^{-1}
a_1	-2,9331 x 10^{-8} Pa^{-1}
a_2	1,1043 x 10^{-10} K^{-1} Pa^{-1}
b_0	5,707 x 10^{-6} K Pa^{-1}
b_1	-2,051 X 10^{-8} Pa^{-1}
C_0	1.9898 x 10^{-4} K Pa^{-1}
C_1	- 2,376 x 10^{-6} Pa^{-1}
d	1,83 x 10-11K^2Pa^{-2}
e	-0,765 x 10^{-8} K^2Pa^{-2}

- **Universal Constant of ideal gases, R**

  R = 8.314510 ± 8,4 x lO-6 J . mol^{-1} . K^{-1}

- **Molar fraction of water steam, Xv**

  In function of relative humidity, h

$$x_v = hf(p,t)\frac{p_{sv}(t)}{p}$$

In function of temperature of dew point, tr

$$X_v = f(p,t_r)\frac{p_{sv}(t_r)}{p}$$

Where:

h	Relative humidity
p_{sv}	Pressure of saturated steam
$f(p,t_r)$	Fugacity factor

Enhancement factor f $f(p,t_r)$

$$f = \alpha + \beta p + \gamma t^2$$

Where:

α	1,000 62
β	3.14 x 10^{-8} Pa^{-1}
γ	5,6 x 10^{-7} K^{-2}
p	Air pressure in Pa
T	Air temperature in ^{O}C or dew point temperature (t_r) in ^{O}C

Pressure of saturated steam, p_{sv}

$$p_{sv} = 1Pa x \exp\left[AT^2 + BT + C + \frac{D}{T}\right]$$

Where:

A 1,237 884 7 x 10-5 K-2
B -1,912 131 6 x 10-2 K-1
C 33,937 110 47
D -6,343 164 5 x 103 K
T Air temperature in K or dew point temperature (Tr) in K

UNCERTAINTY OF AIR DENSITY

SOURCES OF UNCERTAINTY

Atmospheric temperature

$$up = \sqrt{u_{p1}^2 + u_{p2}^2 + u_{p3}^2}$$

- Calibration of barometric

$$u_{p1} = \frac{U_B}{k}$$

- Resolution of barometric

$$u_{p2} = \frac{d_B}{\sqrt{12}}$$

- Variation of atmospheric pressure during calibration

$$up3 = \frac{p^+ - p^-}{\sqrt{24}}$$

Environmental conditions

$$u_t = \sqrt{u_{t1}^2 + u_{t2}^2 + u_{t3}^2}$$

- Calibration of thermometer

$$u_{t1} = \frac{U_t}{k}$$

- Resolution of instrument

$$u_{t2} = \frac{d_t}{\sqrt{12}}$$

- Variation of temperature during calibration

$$u_{t3} = \frac{t^+ - t^-}{\sqrt{24}}$$

Relative humidity of air

$$u_h = \sqrt{u_{h1}^2 + u_{h2}^2 + u_{h3}^2}$$

- Calibration of hygrometer

$$u_{h1} = \frac{U_h}{k}$$

- Resolution of hygrometer

$$u_{h2} = \frac{d_h}{\sqrt{12}}$$

- Variation of the air relative humidity during calibration

$$u_{h3} = \frac{h^+ - h^-}{\sqrt{24}}$$

Constant R of ideal gases

u_R = 84x 10^{-7} J mol^{-1} K^{-1}

Equation adjustment for the determination of air density

$$u_{ec} = (1x10^{-4})(0,9495) = 9,50x10^{-5}\,kgm^{-3}$$

Sensitivity Coefficient

Pressure

$$c_p = \left[\frac{\partial \rho_a}{\partial p} + \left(\frac{\partial \rho_a}{\partial Z} \frac{\partial Z}{\partial p} \right) + \left(\frac{\partial \rho_a}{\partial Z} \frac{\partial Z}{\partial X_v} \frac{\partial X_v}{\partial f} \frac{\partial f}{\partial p} \right) + \left(\frac{\partial \rho_a}{\partial Z} \frac{\partial Z}{\partial X_v} \frac{\partial X_v}{\partial p} \right) + \left(\frac{\partial \rho_a}{\partial X_v} \frac{\partial X_v}{\partial f} \frac{\partial f}{\partial p} \right) + \left(\frac{\partial \rho_a}{\partial Xv} \frac{\partial Xv}{\partial_p} \right) \right]$$

Temperature

$$ct = \left[\begin{array}{l} \left(\frac{\partial \rho_a}{\partial Z} \cdot \frac{\partial Z}{\partial T} \cdot \frac{\partial T}{\partial t} \right) + \left(\frac{\partial \rho_a}{\partial Z} \cdot \frac{\partial Z}{\partial t} \right) + \left(\frac{\partial \rho_a}{\partial Z} \cdot \frac{\partial Z}{\partial X_v} \cdot \frac{\partial X_v}{\partial f} \cdot \frac{\partial f}{\partial t} \right) + \left(\frac{\partial \rho_a}{\partial Z} \cdot \frac{\partial Z}{\partial X_v} \cdot \frac{\partial X_v}{\partial P_{sv}} \cdot \frac{\partial P_{sv}}{\partial T} \cdot \frac{\partial T}{\partial t} \right) \\ + \left(\frac{\partial \rho_a}{\partial T} \cdot \frac{\partial T}{\partial t} \right) + \left(\frac{\partial \rho_a}{\partial X_v} \cdot \frac{\partial X_v}{\partial f} \cdot \frac{\partial f}{\partial t} \right) + \left(\frac{\partial \rho_a}{\partial X_v} \cdot \frac{\partial X_v}{\partial P_{sv}} \cdot \frac{\partial P_{sv}}{\partial T} \cdot \frac{\partial T}{\partial t} \right) \end{array} \right]$$

Relative humidity.

$$C_h = \left[\left(\frac{\partial \rho_a}{\partial Z} \cdot \frac{\partial Z}{\partial X_v} \cdot \frac{\partial X_v}{\partial h} \right) + \left(\frac{\partial \rho_a}{\partial X_v} \cdot \frac{\partial X_v}{\partial h} \right) \right]$$

Where:

$$\frac{\partial T}{\partial t} = 1$$

$$\frac{\partial P_{sv}}{\partial T} = \left[\exp\left(AT^2 + BT + C + \frac{D}{T} \right)\left(2AT + B - \frac{D}{T^2} \right) \right]$$

$$\frac{\partial f}{\partial p} = \beta = 3{,}14x10^{-8}\,Pa^{-1}$$

$$\frac{\partial f}{\partial t} = 2\gamma t$$

$$\frac{\partial X_v}{\partial h} = \frac{fP_{sv}}{p}$$

$$\frac{\partial x_v}{\partial f} = \frac{hP_{sv}}{p}$$

$$\frac{\partial xv}{\partial p} = \frac{-hfP_{sv}}{p^2}$$

$$\frac{\partial X_v}{\partial P_{sv}} = \frac{hf}{p}$$

$$\frac{\partial Z}{\partial p} = \frac{-1}{T}\left[a_0 + a_1 t + a_2 t^2 + (b_0 + b_1 t)X_v + (C_0 + C_1 t)X_v^2\right] + \frac{2p}{T^2}\left(d + ex_v^2\right)$$

$$\frac{\partial Z}{\partial T} = \frac{p}{T^2}\left[a_0 + a_1 t + a_2 t^2 + (b_0 + b_1 t)X_v + (C_o + C_1 t)Xv_2^{}\right] - \frac{2p^2}{T^3}\left(d + ex_v^2\right)$$

$$\frac{\partial Z}{\partial t} = \frac{-p}{T}\left(a_1 + 2a_2 t + b_1 X_v + c1X_v^2\right)$$

$$\frac{\partial Z}{\partial X_v} = \frac{-p}{T}\left(b_0 + b_1 t + 2c_o x_v + 2c_1 t x_v\right) + \frac{2p2eX_v}{T^2}$$

$$\frac{\partial\rho}{\partial p} = \frac{Ma}{ZRT}\left(1 - Xv\left(1 - \frac{Mv}{Ma}\right)\right)$$

$$\frac{\partial\rho}{\partial Z} = \frac{-pM_a}{Z^2RT}\left[1 - x_v\left[1 - \frac{M_v}{M_a}\right]\right]$$

$$\frac{\partial\rho}{\partial T} = \frac{-pM_a}{ZRT^2}\left[1 - X_v\left(1 - \frac{M_v}{M_a}\right)\right]$$

$$\frac{\partial\rho}{\partial X_v} = \frac{-pM_a}{ZRT}\left(1 - \frac{M_v}{M_a}\right)$$

$$\frac{\partial\rho}{\partial R} = \frac{-pM_a}{ZR^2T}\left[-Xv\left(1 - \frac{M_v}{M_a}\right)\right]$$

Constant R

$$C_R = \left(\frac{\partial\rho_a}{\partial R}\right) \qquad C_R = -0{,}114203618\,kgm^3 j^{-1} molK$$

Equation

$$C_{ec} = 1$$

Uncertainty evalation

$$u_p = \sqrt{\sum_t \left[c_i \cdot u(x_i)\right]^2}$$

Degrees of freedom

$$v_{ef} = \frac{u_\gamma^4}{\sum_t^n \frac{u_{xt}^4}{v_t}} = \frac{u_\rho^4}{\frac{u_p^4}{v_p} + \frac{u_t^4}{v_t} + \frac{u_h^4}{v_h} + \frac{u_R^4}{v_R} + \frac{u_{ec}^4}{v_{ec}}}$$

Expanded uncertainty

$$U = k \cdot u_p$$

: Consider that the atmospheric pressure, temperature and relative humidity are correlated, its uncertainty:

$$u_p = \sqrt{\sum_i \left(c_i \cdot u(x_i)\right)^2 + 2\sum_{i=1}^{N-1} \sum_{j(i+1)}^{N} c_i c_j u(x_i) u(x_j) r(x_i, x_j)}$$

Where :

$$r(t,p) = \frac{\frac{1}{n(n-1)} \sum_{k=1}^{n} \left(t_k - \bar{t}\right)\left(p_k - \bar{p}\right)}{s(\bar{t})s(\bar{p})}$$

$$r(p,h) = \frac{\frac{1}{n(n-1)} \sum_{k=1}^{n} \left(p_k - \bar{p}\right)\left(h_k - \bar{h}\right)}{s(\bar{p})s(\bar{h})}$$

$$r(t,h) = \frac{\frac{1}{n(n-1)} \sum_{k=1}^{n} \left(t_k - \bar{t}\right)\left(h_k - \bar{h}\right)}{s(\bar{t})s(\bar{h})}$$

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Thanks for the attention